Math 204

Homework 2.1

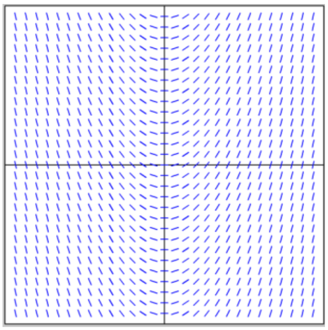
1) Which of the following DE has the direction field shown in the figure?

   a) $\dfrac{dy}{dx} = x^2 - y^2$

   b) $\dfrac{dy}{dx} = x$

   c) $\dfrac{dy}{dx} = -2y$

   d) $\dfrac{dy}{dx} = \dfrac{x}{y}$



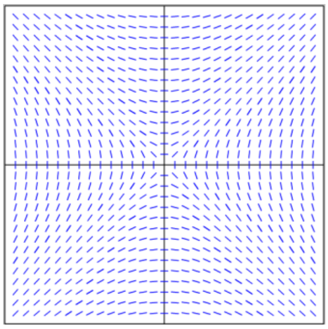
2) Which of the following DE has the direction field shown in the figure?

   a) $\dfrac{dy}{dx} = x^2 - y^2$

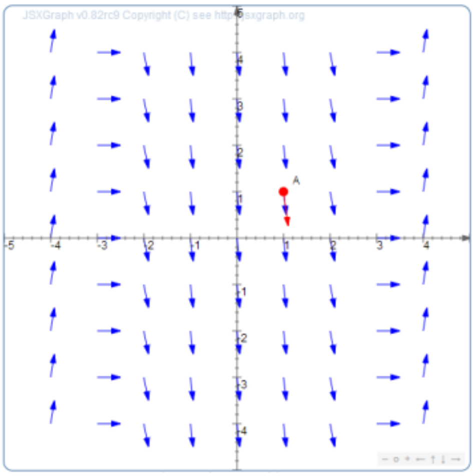
   b) $\dfrac{dy}{dx} = x$

   c) $\dfrac{dy}{dx} = -2y$

   d) $\dfrac{dy}{dx} = \dfrac{x}{y}$



3) Use the following direction field to for the differential equation $\dfrac{dy}{dx} = f(x, y)$ to identify where the function is positive, negative or zero. Is $f(x,y)$ a function of x alone, y alone, or a function of both variables together? Find a function $f(x,y)$ whose vector field looks like this.



4) Draw the vector field for $\frac{dy}{dx} = y - 1$ and sketch an appropriate solution curves passing through the points

a) (0,0)

b) (1,2).

5) Find the critical points and draw the phase portrait of the given autonomous differential equations. Classify each critical point as asymptotically stable, unstable, or semi-stable. Sketch the equilibrium solutions and typical solution curves in the different regions in the *x*-*y* plane.

a) $\frac{dy}{dx} = y^3 - y^2$

b) $\frac{dy}{dx} = y^3 - 4y$

c) $\frac{dy}{dx} = y^2 - 3y - 10$

Homework 2.2

1) In problems 1-5 determine whether the given differential equation is separable.

   1) $\frac{dy}{dx} = \frac{y^2+y}{x^2+x}$

   2) $\frac{dy}{dx} = \frac{1}{x(x-y)}$

   3) $x\frac{dy}{dx} = ye^{x/y} - x$

   4) $\frac{y}{x^2}\frac{dy}{dx} + \cos(x+y) = 0$

   5) $(x+4)dy = (x^2y - 8 + 4x^2 - 2y)dx$

2) In problems 6-10 solve the given differential equation by separation of variables.

   6) $\frac{dz}{dw} = we^{3w+2z}$

   7) $x\sin^2 y\ \frac{dy}{dx} = (x+1)^2$

   8) $(x + xy^2)dx + e^{x^2}ydy = 0$

   9) $(x+1)^2 \ln y\frac{dy}{dx} = \frac{x}{y^2}$

   10) $x\frac{dy}{dx} = \frac{x^2-x-2}{xy+x+y+1}$

3) In problems 11-13solve the given initial value problem.

   11) $\frac{dy}{dx} = 3x^2e^{-y}, \qquad y(0) = 1$

   12) $\frac{dy}{dx} = \frac{y}{x(x+1)}, \qquad y(1) = 3$

   13) $y' = 2x\cos^2 y, \qquad y(0) = \pi/4$

4) Proceed as in example 5 to find an explicit solution of the given initial value problem.

$$\frac{dy}{dx} = y^2 \sin x^2, \qquad y(-2) = \frac{1}{3}$$

Homework 2.3

I. In problems 1-6 determine whether the given equation is separable, linear, neither, or both.

1) $x^2 \frac{dy}{dx} + \cos x = \mathrm{y}$
2) $\frac{dx}{dt} + xt = e^x$
3) $x\frac{dx}{dt} + t^2 x = \sin t$
4) $3t = e^t \frac{dy}{dt} + y \ln t$
5) $(t^2+1)\frac{dy}{dt} = yt - t$
6) $3r = \frac{dr}{d\theta} - \theta^3$

II. In problems 7-17 find the general solution of the given differential equation. Give the largest interval *I* over which the solution is defined. Determine whether there are any transient terms in the general solution.

7) $\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = x\cos x$
8) $y' + y = \sqrt{1+\cos 2x}$
9) $\frac{dr}{d\theta} + r\tan\theta = \sec\theta$
10) $\frac{dy}{dx} = \frac{y}{x} + 2x + 1$
11) $(t+y+1)dt - dy = 0$
12) $\frac{dy}{dx} = x^2 e^{-3x} - 4y$
13) $4x^3 y + x^4 y' = \sin^3 x$
14) $ydx - 2(x+y^4)dy = 0$
15) $(x^2+1)\frac{dy}{dx} = x^2 + 2x - 1 - 4xy$
16) $\cos^2 x \sin x \frac{dy}{dx} + (\cos^3 x)y = 1$

III. In problems 17-19 solve the given initial value problem.

17) $xy' + y = e^x, \qquad y(1) = 2$
18) $\sin x \frac{dy}{dx} + y\cos x = x\sin x, \qquad y\left(\frac{\pi}{2}\right) = 2$
19) $(x+1)\frac{dy}{dx} + y = \ln x, \qquad y(1) = 10$

5) In problem 20 proceed as in example 6 to solve the initial value problem

20) $\frac{dy}{dx} + 2xy = f(x), \quad y(0) = 2,$ where $f(x) = \begin{cases} x & 0 \le x < 1 \\ 0 & x \ge 1 \end{cases}$

Homework 2.4

I. In problems 1-7 classify the equation as separable, linear, exact, or none of these. Some equations may have more than one classification.

1) $(x^2y + x^4\cos x)dx - x^3dy = 0$
2) $xydx + dy = 0$
3) $\frac{dy}{dx} = \frac{x-y}{x}$
4) $[2x + \cos(xy)]dx + [\sin(xy) + 2y]dy = 0$
5) $y^2dx + (2xy + \cos y)dy = 0$
6) $xyy' = 2xe^y$
7) $(ye^{xy} + 2x)dx + (xe^{xy} - 2y)dy = 0$

II. In problems 8-14 determine whether the differential equation is exact. If it is exact solve it.

8) $(2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$
9) $(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$
10) $(1 + e^xy + xe^xy)dx + (xe^x + 2)dy = 0$
11) $\left(\frac{t}{y}\right)dy + (1 + \ln y)dt = 0$
12) $(\tan x - \sin x\sin y)dx + \cos x\cos y\,dy = 0$
13) $\left(\frac{1}{y}\right)dx - \left(3y - \frac{x}{y^2}\right)dy = 0$
14) $[\frac{2}{\sqrt{1-x^2}} + y\cos(xy)]dx + [x\cos(xy) - y^{-\frac{1}{3}}]dy = 0$

III. In problem 15 and 16 find the value of *k* so that the equation is exact

15) $(kx^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$
16) $(6xy^3 + \cos y)dx + (2kx^2y^2 - x\sin y)dy = 0$

IV. In problems 17-19 solve the given differential equation by finding an appropriate integrating factor as in example 4.

17) $(2x^2 + y)dx + (x^2y - x)dy = 0$
18) $(y^2 + 2xy)dx - x^2dy = 0$
19) $\cos x\,dx + \left(1 + \frac{2}{y}\right)\sin x\,dy = 0$

Homework 2.5

I. In problems 1-9 solve the following differential equations using an appropriate substitution.

1) $\frac{dy}{dx} = y - x - 1 + (x - y + 2)^{-1}$

2) $(xy + y^2 + x^2)dx - x^2 dy = 0$

3) $\frac{dy}{dx} - 5y = -\frac{5}{2}xy^3$

4) $-ydx + \left(x + \sqrt{xy}\right)dy = 0$

5) $\frac{dy}{dx} = \frac{1 - x - y}{x + y}$

6) $\frac{dr}{d\theta} = \frac{r^2 + 2r\theta}{\theta^2}$

7) $\cos(x + y)dy = \sin(x + y)dx$

8) $3(1 + t^2)dy = 2ty(y^3 - 1)dt$

9) $xdy - y(\ln y - \ln x + 1)dx = 0$

II. In problems 10-12solve the following initial value problem.

10) $\left(x + ye^{y/x}\right)dx - xe^{y/x}dy = 0, \qquad y(1) = 0$

11) $\frac{dy}{dx} = \cos(x + y), \qquad y(0) = \pi/4$

12) $\frac{dy}{dx} = \frac{3x + 2y}{3x + 2y + 2}, \qquad y(-1) = -1$

III. Find a one-parameter family of solutions for the differential equation

$$\frac{dy}{dx} = x^3(y - x)^2 + \frac{y}{x}$$

Where $y_1 = x$ is a known solution of the equation.